General Certificate of Education
Advanced Level Examination
January 2012

Mathematics MM2B

Unit Mechanics 2B

Wednesday 25 January 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 A plane is dropping packets of aid as it flies over a flooded village. The speed of a packet when it leaves the plane is $60\,\text{m}\,\text{s}^{-1}$. The packet has mass 25 kg.

The packet falls a vertical distance of 34 metres to reach the ground.

(a) Calculate the kinetic energy of the packet when it leaves the plane. *(2 marks)*

(b) Calculate the potential energy lost by the packet as it falls to the ground. *(2 marks)*

(c) Assume that the effect of air resistance on the packet as it falls can be neglected.

(i) Find the kinetic energy of the packet when it reaches the ground. *(2 marks)*

(ii) Hence find the speed of the packet when it reaches the ground. *(2 marks)*

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2 A particle, of mass 50 kg, moves on a smooth horizontal plane. A single horizontal force

$$[(300t - 60t^2)\,\mathbf{i} + 100\text{e}^{-2t}\,\mathbf{j}] \text{ newtons}$$

acts on the particle at time t seconds.

The vectors **i** and **j** are perpendicular unit vectors.

(a) Find the acceleration of the particle at time t. *(2 marks)*

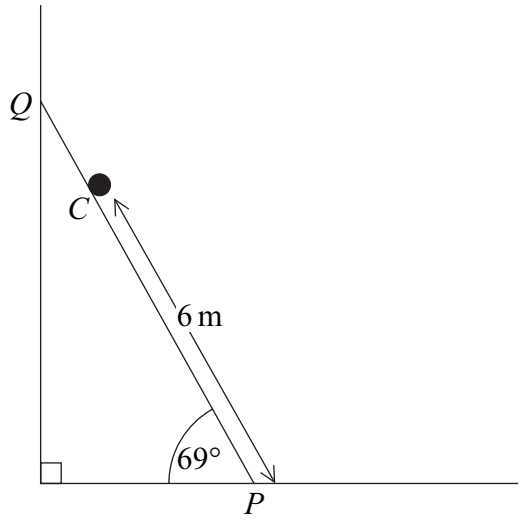
(b) When $t = 0$, the velocity of the particle is $(7\mathbf{i} - 4\mathbf{j})\,\text{m}\,\text{s}^{-1}$.

Find the velocity of the particle at time t. *(4 marks)*

(c) Calculate the speed of the particle when $t = 1$. *(4 marks)*

3 A uniform ladder PQ, of length 8 metres and mass 28 kg, rests in equilibrium with its foot, P, on a rough horizontal floor and its top, Q, leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall and the angle between the ladder and the floor is 69°.

A man, of mass 72 kg, is standing at the point C on the ladder so that the distance PC is 6 metres. The man may be modelled as a particle at C.



(a) Draw a diagram to show the forces acting on the ladder. *(2 marks)*

(b) With the man standing at the point C, the ladder is on the point of slipping.

(i) Show that the magnitude of the reaction between the ladder and the vertical wall is 256 N, correct to three significant figures. *(4 marks)*

(ii) Find the coefficient of friction between the ladder and the horizontal floor. *(4 marks)*

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4 A car travels along a straight horizontal road. When its speed is $v\,\text{m s}^{-1}$, the car experiences a resistance force of magnitude $25v$ newtons.

(a) The car has a maximum constant speed of $42\,\text{m s}^{-1}$ on this road.

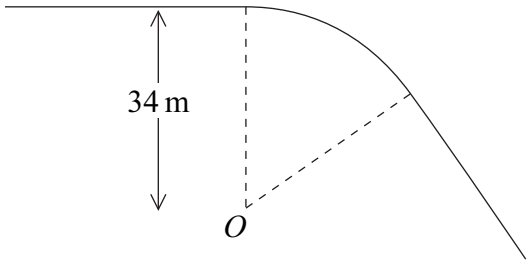
Show that the power being used to propel the car at this speed is 44 100 watts. *(2 marks)*

(b) The car has mass 1500 kg.

Find the acceleration of the car when it is travelling at $15\,\text{m s}^{-1}$ on this road under a power of 44 100 watts. *(4 marks)*

5 A parcel is placed on a flat rough horizontal surface in a van. The van is travelling along a horizontal road. It travels around a bend of radius 34 m at a constant speed. The coefficient of friction between the parcel and the horizontal surface in the van is 0.85 .

Model the parcel as a particle travelling around part of a circle of radius 34 m and centre O, as shown in the diagram.



Find the greatest speed at which the van can travel around the bend without causing the parcel to slide. *(6 marks)*

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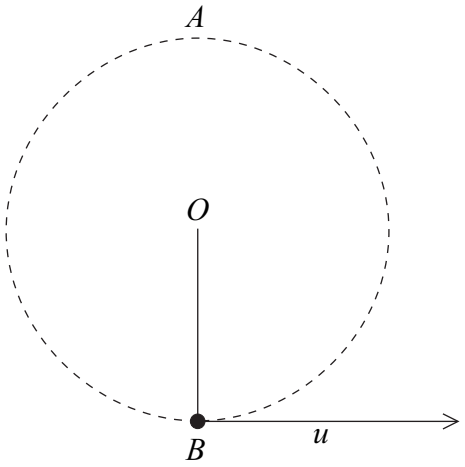
6 Alice places a toy, of mass 0.4 kg, on a slope. The toy is set in motion with an initial velocity of $1\,\text{m}\,\text{s}^{-1}$ down the slope. The resultant force acting on the toy is $(2 - 4v)$ newtons, where $v\,\text{m}\,\text{s}^{-1}$ is the toy's velocity at time t seconds after it is set in motion.

(a) Show that $\dfrac{\text{d}v}{\text{d}t} = -10\,(v - 0.5)$. *(2 marks)*

(b) By using $\displaystyle\int \frac{1}{v - 0.5}\,\text{d}v = -\int 10\,\text{d}t$, find v in terms of t. *(5 marks)*

(c) Find the time taken for the toy's velocity to reduce to $0.55\,\text{m}\,\text{s}^{-1}$. *(3 marks)*

7 A small bead, of mass m, is suspended from a fixed point O by a light inextensible string of length a. With the string taut, the bead is at the point B, vertically below O, when it is set into vertical circular motion with an initial horizontal velocity u, as shown in the diagram.



The string does not become slack in the subsequent motion. The velocity of the bead at the point A, where A is vertically above O, is v.

(a) Show that $v^2 = u^2 - 4ag$. *(2 marks)*

(b) The ratio of the tensions in the string when the bead is at the two points A and B is $2:5$.

(i) Find u in terms of g and a. *(7 marks)*

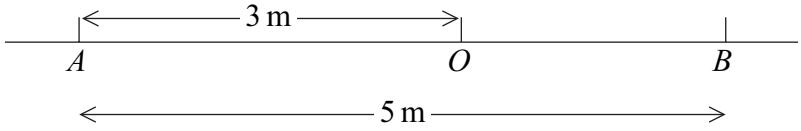
(ii) Find the ratio $u:v$. *(2 marks)*

8 An elastic string has one end attached to a point O fixed on a rough horizontal surface. The other end of the string is attached to a particle of mass 2 kg. The elastic string has natural length 0.8 metres and modulus of elasticity 32 newtons.

The particle is pulled so that it is at the point A, on the surface, 3 metres from the point O.

(a) Calculate the elastic potential energy when the particle is at the point A. *(3 marks)*

(b) The particle is released from rest at the point A and moves in a straight line towards O. The particle is next at rest at the point B. The distance AB is 5 metres.



Find the frictional force acting on the particle as it moves along the surface. *(6 marks)*

(c) Show that the particle does not remain at rest at the point B. *(2 marks)*

(d) The particle next comes to rest at a point C with the string slack.

Find the distance BC. *(2 marks)*

(e) Hence, or otherwise, find the total distance travelled by the particle after it is released from the point A. *(1 mark)*